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AUTHOR	Phillips, Harry L.; Kluttz, Marguerite
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ABSTRACT

This guide for parents explains the objectives of the modern mathematics being taught in the schools and discusses the teaching methods being used. A few of the elementary concepts of modern mathematics (number lines, searching for patterns, different ways of analyzing problems, number bases, and sets) are briefly explained and justifications are given for their inclusion in the modern mathematics curriculum. Short descriptions of the Madison Project, Suppes' Projects, University of Illinois Committee on School Mathematics (UICSM), School Mathematics Study Group (SMSG), the College Entrance Examination Board (CEEB) Commission's Recommendations, the Ball State Program, and the University of Maryland Mathematics Project are given along with a brief explanation of the National Defense Education Act (NDEA), Title III. Eleven suggestions of ways parents may help their children in learning modern mathematics and a reading list of 14 books are included in the final sections. (DT)

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Modern Mathematics and Your Child

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Modern Mathematics and Your Child

By
Harry L. Phillips
Specialist, Mathematics
and
Marguerite Kluttz
Research Assistant, Mathematics
Instructional Resources Branch

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JOHN W. GARDNER
Secretary
Office of Education
FRANCIS KEPPEL
Commissioner

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Courtesy: Oklahoma City Public Schools

The modern learning of elementary mathematics by the meaningful method.

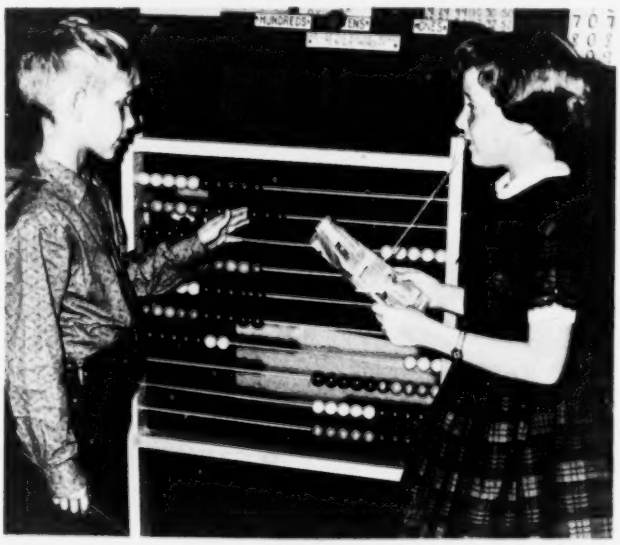
A New Trend and a New Vocabulary

HAVE YOU RECENTLY HEARD your youngster speak of "sets," "the binary number system," "structural patterns," "inverse operations," or one of many other mathematical-sounding words or phrases? If you have, don't be alarmed. He isn't cultivating a new teenage lingo; he is talking about modern mathematics. If you are not yet one of the parents who have experienced this, you may as well prepare yourself now! The trend toward a more meaningful, dynamic, exciting, instructional program in mathematics is well established and is gaining more and more momentum as time goes on.

Are you interested in knowing more about this sweeping movement? Would you like to be in on some of the fascinating aspects of mathematics which in your own school days you probably missed? Do you want to know why these ideas and topics are being included in up-to-date mathematics instructional programs? Do you want to know what you as a parent can do to enhance your youngster's learning and study of mathematics? It is the aim of this publication to provide answers to these and related questions.

First, let us define a few terms and generalize a little so that the material which follows will be more readily and clearly understandable. The word "mathematics" as it is used in this publication means much more than arithmetic, algebra, and geometry; it represents a way of thinking and reasoning, a systematized body of knowledge which will provide invaluable assistance in determining whether ideas or statements are true or what is the probability of their being true. It represents a universal language which could be recognized by rational beings wherever they may exist. It is for this reason that the radio astronomers who are making a conscientious attempt to contact life elsewhere in the universe send their contact messages out in mathematical code. It is true that mathematics in its applications also solves

complex problems of industry, Government, and defense, but this is only one small part of a much more comprehensive world—the world of mathematics.



Courtesy: Wichita, Kansas, Public Schools

These youngsters are involved in the study of mathematics in a way which is quite different from the rote memorization method.

THE NEIGHBORS **By George Clark**

"Dad! I can work these problems the way you're doing them. Counting on your fingers!"

Courtesy: New York News Syndicate, Inc.

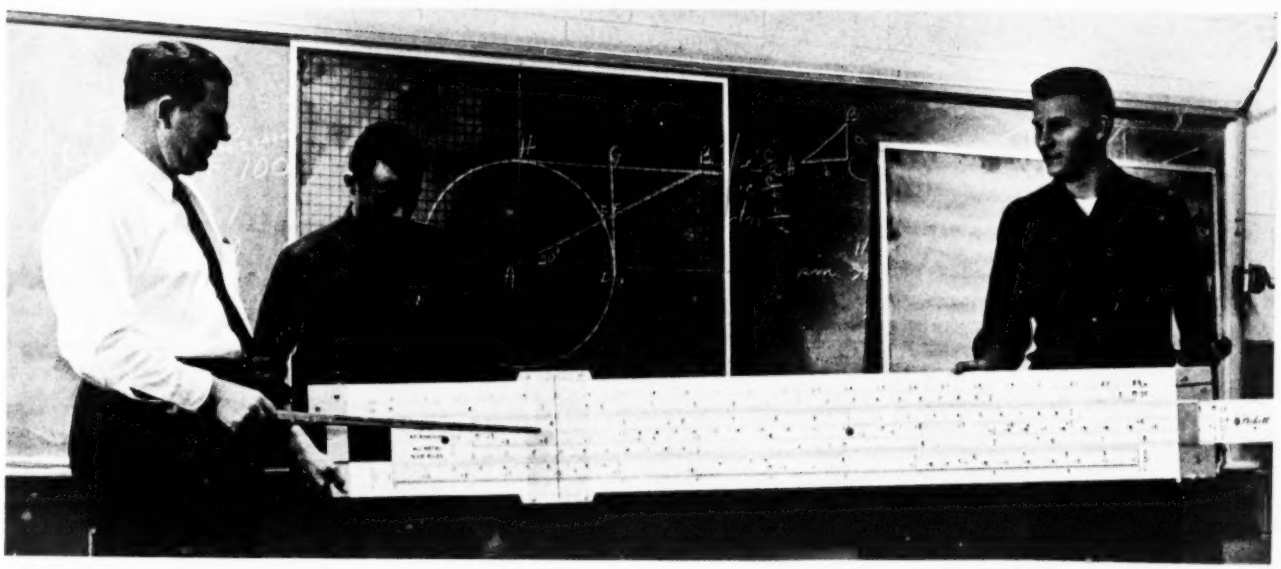
What Is Modern Mathematics?

New Methods and New Directions

Modern mathematics, as the term is popularly used, does not necessarily mean that the mathematical concepts or notions involved have been recently devised or conceived. Many of the ideas which will be discovered by your youngster in his study will be referred to as "modern mathematics," although in fact they are hundreds of years old; they are modern only in the sense that they are new or modern in the school instructional program; that is, in the sense that they are for the first time being presented to students at this particular grade or course level. A better description of the situation would be "a modern program of mathematics for the schools." However, this publication follows the popular trend and refers to the new instructional developments as "modern mathematics." This, in all probability, is the way you will hear the term used in the PTA, by your youngster's mathematics teacher, or by the school principal.

Your first reaction to some of the ideas which follow may well be: "That isn't the way I learned arithmetic," or "This wasn't mentioned by my arithmetic teacher." Both of these statements would probably be true, but this does not mean that all these ideas have been discovered since you were in school. It simply means that times have changed; a different method of teaching arithmetic or mathematics in general is now being employed—additional ideas and topics are being included in the courses.

Does this mean that the mathematics which you studied is no longer applicable or worth while? No, certainly not. The basic factual material of mathematics has not changed. Six times eight is still forty-eight in the common base-ten system. The old or traditional mathematics has not been replaced. Nowadays your youngster will study almost all the elementary mathematics you studied as well as many additional topics.



Courtesy: Bowden School, Carroll County, Ga., Public Schools

Learning aids which are found in some American mathematics classrooms today.

Objectives of the Modern Methods

1. *A proper balance between reasoning and drill.*—One of the basic objectives of mathematics instruction today is to place in proper balance the student's memorization of methods, rules, and facts and, on the other side of the scale, his reasoning power. Although computation and skill development (which you may call drill) are still needed and taught, the emphasis in the new look in mathematics is on the student's discovery and appreciation of structure, patterns, and continuity. In other words, the *why* of computing is now being strongly stressed. For, if the pupil has a better understanding of the basic principles involved and uses a more meaningful approach, perhaps he will be able to cut down appreciably the amount of time he would otherwise have spent on drill exercises. Moreover, the automatic computing machinery now available and being used for numerical work has largely replaced the pencil-and-paper method of calculating.

2. *Adjustment of pace and content to rapid expansion of scientific knowledge.*—Another reason certain mathematical concepts are being taught much earlier than they were in the past is that the body of scientific knowledge characteristic of our culture may be expected to double in one decade. Perhaps you have heard this referred to as the "explosion of knowledge." This means that the content and pace of mathematics courses must also undergo considerable change and adjustment, to enable mathematics to maintain and spearhead the rapidly accelerating scientific development.

Fortunately, as one writer has pointed out, many of the basic concepts of mathematics are inherently simple—so simple that a child can understand them. Moreover, children seem to be naturally attuned to mathematical abstraction, perhaps because it is verging on pure fantasy. (One of the best-loved children's books, *Alice's Adventures in Wonderland*, was written by a professional mathematician whose pen name was Lewis Carroll, and much of its fantasy represents abstruse mathematical ideas.) For example, some of the most difficult problems in current mathematics have arisen from the theory of infinite sets; that is, of collections of an infinite number of things, such as all the points on a line or all the conceivable whole numbers. Yet the late Edward Kasner of Columbia University used to lecture on infinite sets to kindergarten classes. You may have read the book *Mathematics and the Imagination* which he and James Newman wrote. The children, Dr. Kasner found, readily reconciled themselves to the notion of infinity and got the fundamental ideas of the theory of sets faster than some of his undergraduate students.

3. *Increased pleasure in learning.*—Another objective of the new approach is to relieve the feeling of boredom and apathy that used to beset the pupil during the rote memory phase of mathematical study. There is increasing evidence that young people can enjoy learning mathematics. This might be stated in another way—mathematics has certain appealing and highly motivating qualities if taught in a manner that does not conceal these aspects of the subject. Yes, there is meaning in the statement that mathematics should be taught for the sake of mathematics. As students learn the fundamental principles with real understanding and come to realize the relationships between operations like addition and multiplication, they develop an appreciation and feeling for the orderly structure and logic of mathematics. Jerome S. Bruner, Harvard psychology professor, in his book *The Process of Education*, states that the teaching of specific topics or skills without a broader fundamental knowledge of the structural aspects of the discipline can be very dull to the learner and uneconomical to the teacher. This statement is particularly applicable to the teaching and learning of mathematics. Indeed, two of the most important things to be learned in mathematics are the pleasure and ease of the manipulation of abstract concepts and how to understand these ideas in the mathematical setting of structure and logic. The new developments have done much to remove the monotony which may have plagued you in your own youthful experience with mathematics.

Your youngster is fortunate to be growing up in an age in which mathematics education is revitalizing and updating itself. He will discover that there is more than one geometry. He will discover that there are number systems instead of a number system. He will find that a close correlation exists between algebra and geometry. He will develop a mathematical language, somewhat different from yours, but more precise and more complete. He will learn of the significant contributions made by the high-speed electronic computing devices and will gain some understanding of the mathematical operation of these machines. This is certainly a fascinating time for your youngster to be studying mathematics.

A Glimpse of Modern Mathematics

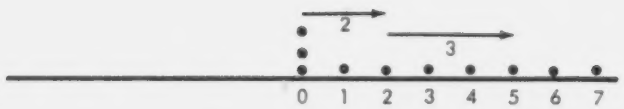
YOUR CHILD may proudly show you that he has mastered what he considers an important principle in his elementary arithmetic class, namely that $3+2=5=2+3$ or that $2\times3=3\times2$. Your immediate reaction may be "So what? I learned those facts without any ado whatsoever." In all probability, however, your youngster is beginning the study of a very important principle which he will later call the commutative law. This may not seem of much significance to either you or the child at this point. But the thorough understanding of a few fundamental principles of arithmetic, such as the commutative, associative, and distributive laws, the closure property, and order will assist him immeasurably when he is later confronted with algebra and higher mathematics courses.

The pupil's understanding of the meaning of addition, subtraction, multiplication, division, fractions, and negative numbers increases considerably when he has the opportunity to explore these ideas with the help of the many intriguing teaching aids now in use, for example, the number line, the abacus, the counting frame, magnetic discs and boards, and Cuisenaire rods.

Use of the Number Line

Example 1: Addition.—To give you some idea of how helpful these aids are, consider the following simple situation. You wish to understand better how addition and subtraction are related. Take the simple problem $2+3=\square$. You would probably read this as asking "Two plus three equals what number?" Your child may say "Two plus three equals a placeholder," or "What is another name for two plus three?" Try this problem on the number line as follows:

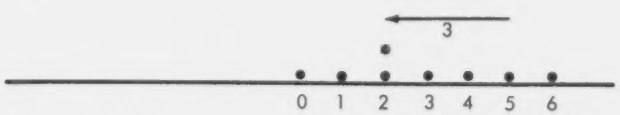
Begin at the point named 0 and go to the right a distance equal to the first addend, namely 2.



Next begin at this point called 2 and add another distance equal to the second addend, namely 3.

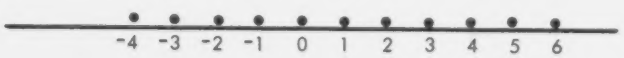
This of course brings you to the point named 5. Therefore, $2+3=5$. Then, 5 is another name for $2+3$.

Example 2: Subtraction.—Next consider starting at 5 and subtracting the 3. If, in adding you traveled along the number line to the right, which would be the logical direction for travel in subtracting?

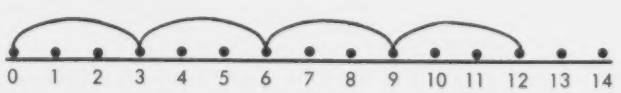


Of course, when 3 is subtracted from 5 there remains the answer 2. Therefore, 2 is another name for $5-3$. You are now beginning to see what is meant by inverse operations. This gives you a picture of the relationship between addition and subtraction. Your child will have many such experiences with the multisensory aids and devices which are part of a modern elementary mathematics program.

Next, extend the number line to the left to include the negative numbers. Try if you can see with new eyes the methods of adding and subtracting the negative numbers.



Example 3: Division and multiplication.—Number lines may also be used to give more meaning to multiplication and division processes. For example, consider the number sentence: $4\times3=n$. What is another name for 4×3?



The obvious answer is 12. Why were 4 steps taken down the number line? How large was each step? Is there any relation between this multiplication process and addition?

What Is a Pattern in Mathematics?

Consider the following relationships:

$$1 = 1 = 1\times 1$$
$$1+3 = 4 = 2\times 2$$
$$1+3+5 = 9 = \ ?$$
$$1+3+5+7 = 16 = \ ?$$
$$1+3+5+7+9 = 25 = \ ?$$

Can you give the correct answers?

$1\times 9 = 9$
$2\times 9 = 18$ but $1+8=9$
$3\times 9 = 27$ but $2+7=9$
$4\times 9 = 36$ but $3+6=9$
$5\times 9 = 45$ but $4+5=9$
$6\times 9 = \ ?$ but $5+4=\ ?$

Can you give the correct answers?

The two exercises above may appear at first glance to be tricks but, in fact, they contain a deeper significance. Your youngster may be doing many problems of this type. This will mean that he is cultivating a feeling for exploring and discovering, and at the same time gaining an appreciation for a well-designed and orderly number system full of interesting aspects to be investigated. Moreover, as time goes on, he will be able to explain why such patterns exist in a number system. There are many things about our number system of which you may not be aware. For example, the study of pattern and structure in the modern program will systematically treat the natural numbers, zero, the negative integers, the rational and irrational numbers, and, finally, the complex numbers. The student will discover that other number systems (besides our common base ten system) also possess certain characteristic patterns. And during all this he will be gaining valuable insight into the innermost intricacies of number system patterns which will lead him later to an understanding of algebraic structure.

Different Ways of Analyzing Problems

On certain types of problems your youngster may use strange-looking methods of attack. The special process used in solving a problem in mathematics is known as an algorithm, but a particular problem may be solved by more than one algorithm. Some examples follow:

Example 1: Subtraction.—$17-9$ indicates that 9 is to be subtracted from 17. One youngster may express himself in this way.

17−9=? Another may use this arrangement:

$$\begin{array}{r} 10+7 \\ -9 \\ \hline 1+7=8 \end{array}$$

17−9=
7+10−9=
7+(10−9)=
7+1=8

These are but two of the various methods of attack that could be chosen to show the operational procedure.

Example 2: Simple multiplication.—Now take the multiplication exercise. This is probably the method you used for multiplying 78×34:

$$\begin{array}{r} 78 \\ \times 34 \\ \hline 312 \\ 234 \\ \hline 2652 \end{array}$$

Did you understand why you placed the 4 of 234 in line with (under) the 3 in the multiplier 34 above?

You may see your youngsters using various multiplication algorithms, such as:

$$\begin{array}{r} 78 \\ \times 34 \\ \hline 70\times\ 4=\ 280 \\ 8\times\ 4=\ 32 \\ 70\times 30=2100 \\ 8\times 30=\ 240 \\ \hline 2652 \end{array} \qquad \begin{array}{r} \\ \\ 312 \\ \\ 2340 \\ \hline 2652 \end{array}$$

Or this:

78×34
78×(30+4)
78(30)+78(4)
30(78)+4(78)
30(70+8)+4(70+8)
30(70)+30(8)+4(70)+4(8)
(2100+240)+(280+32)
2340+312
2652

These operations clearly show that 234 in your method above was really 2340.

But isn't this a lot of extra work? Remember you didn't learn it this way—and the chances are that you didn't understand it either. You were given a rule to follow and no opportunity for using your imagination.

Example 3: Short form of division.—Take the division algorithm. These methods may be more meaningful to some youngsters than the usual short division form:

$$6\overline{)18} = 6\overline{)12+6} \qquad \begin{array}{r} 2+1=3 \end{array}$$

$$8\overline{)1136}$$

800 ← (8 times 100) —— 100
336
320 ← (8 times 40) —— 40
16
16 ← (8 times 2) —— 2
142

This method makes clear that we are actually multiplying the 8 by 1 hundred, 4 tens, and 2 ones.

Example 4: Long division.—With larger numbers a separation process is being used. In dividing 4644 by 54, the child is asked to make an intelligent guess and write it to the side or above as follows:

					86
					6
					80
54/4644			or	54/4644	
4320	80			4320	
324				324	
324	6			324	
	86				

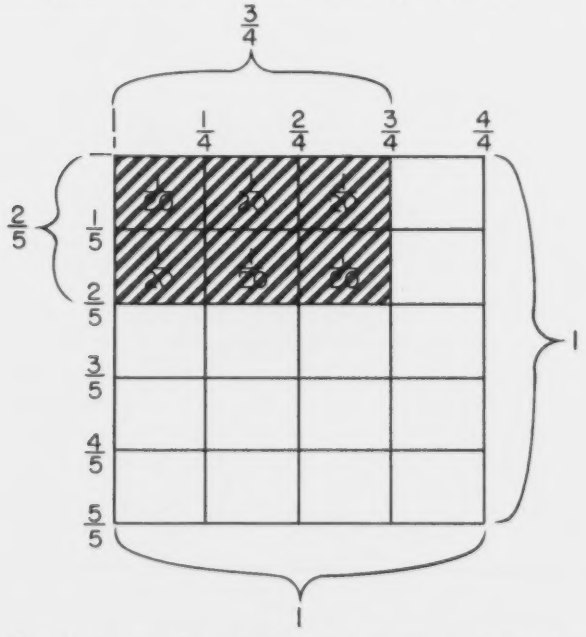
He repeats this process until the remainder is less than 54. Then the guesses are added to find the quotient, which is the answer. Of course, all youngsters will not make the same guesses, but they should finish with the same result:

54/4644	
3780	70
864	
540	10
324	
324	6
	86

As the youngster advances in experience, his guesses should improve and he will look for ways of perfecting them.

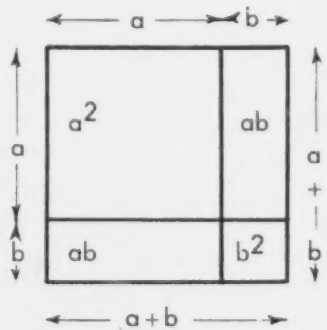
Are these not rather interesting methods of doing subtraction, multiplication, and division? Remember you are not particularly concerned with speed in this work; you are more vitally concerned with understanding the true meaning of the operations themselves.

Example 5: Multiplication of fractions.—Consider the multiplication of two fractions such $\frac{3}{4} \times \frac{2}{5} = n$



Count the number of shaded squares, each of which is $\frac{1}{20}$. Does it now seem clear that $\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$?

Example 6: Algebra.—This approach may also be used to answer more advanced questions such as: $(a+b)^2 = \_\_\_\_\_$.



Therefore $(a+b)^2 = a^2 + 2ab + b^2$

Don't you agree the preceding examples are rather effective demonstrations of processes or facts that were formerly taught through memorization only? The student involved in a modern instructional program of mathematics is doing many of these kinds of problems.

When Does 1+1=10?

The decimal (base-ten) system.—Probably our practice of representing all numbers by combinations of the ten digits—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9—had its origin in the fact that people counted by using the ten fingers on the hands. This counting set is the number base of the decimal numeration system. In the decimal system, as you know, 3254 stands for $(3\times10^3)+(2\times10^2)+(5\times10^1)+(4\times10^0)$. (The zero power of any number is 1—or $10^0=1$, the first power of a number is the number itself—or $10^1=10$, $10^2=10\times10=100$, and $10^3=10\times10\times10=1000$.) This is the place value notation which appears in many textbooks:

$3\times10^3=3\times1000=$ 3 Thousands
$2\times10^2=2\times100\ =$ 2 Hundreds
$5\times10^1=5\times10\ \ =$ 5 Tens
$4\times10^0=4\times1\ \ \ =$ 4 Ones

Courtesy: Kaunod Elementary School, Spreckelsville, Maui, Hawaii

Preparing to compute in the nondecimal system, using base eight.

The binary (base-two) system.—But youngsters in modern mathematics classes are learning other numeration systems. Many of these have special uses. The time of day is calculated on a 12-hour or a 24-hour base; the hour on a 60-minute base; the week on a 7-day base, and so forth. Electronic digital computers do arithmetic in the binary system, which uses only two symbols, namely, 0 and 1 (base two). The 1 can be represented by an electronic circuit with the switch on, and the 0 by the same circuit with the switch off. Therefore, it is easy for an electronic circuit to count effectively in the base-two system. In the binary or base-two system, 1101 stands for (from left to right) $(1\times2^3)+(1\times2^2)+(0\times2^1)+(1\times2^0)$. (Note how this resembles the base-ten system above.) Addition of the terms shows that this becomes $(1\times8)+(1\times4)+(0\times2)+(1\times1)$ or $8+4+0+1$ or 13 in the decimal system. The place value notation of 1101 to the base two is expressed as follows: This is the number given above

1	1	0	1
Eights	Fours	Twos	Ones

The smaller the base, the more symbols it takes to represent a given number.

The counting numbers of the base-ten are:

0 1 2 3 4 5 6 7 8 9 10 – – –

The counting numbers of the base-five are:

0 1 2 3 4 10 11 12 13 14 20 – – –

The counting numbers of the base-two are:

0 1 10 11 100 101 110 111 1000 1001 1010 – – –

Addition and multiplication facts of the base-two numbers are given in these tables:

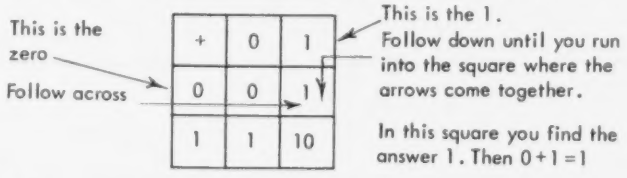
Addition Facts

+	0	1
0	0	1
1	1	10

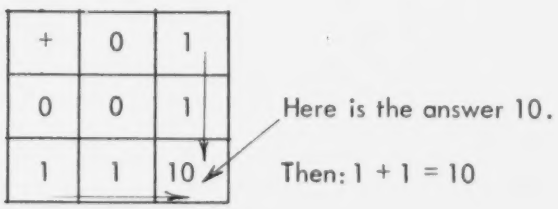
Multiplication Facts

×	0	1
0	0	0
1	0	1

For example, suppose you are going to add 0 and 1 (base two), look at the table which gives you the addition facts, find 0 in the lefthand column and follow this across (see arrow).


This is the zero
Follow across
This is the 1.
Follow down until you run into the square where the arrows come together.
In this square you find the answer 1. Then 0 + 1 = 1

Now do the same for 1+1=?
Remember to find 1 in the first column and follow this row across until you run into the column coming down from 1 in the top row.



This also gives you the answer to the question at the beginning of this explanation: When does $1+1=10$? It is, of course, when adding in the base-two system.

The multiplication table given above works the same way. The multiplication facts of base two are simple: $0\times0=0$, $0\times1=0$, $1\times0=0$, and $1\times1=1$.

Now you should be ready to make some comparisons between the base-ten and the base-two number systems. Can you answer the following questions?

1. Which system requires more symbols to write numbers greater than one?
2. Which system has the fewer different symbols?
3. Which system has fewer addition and multiplication facts?

Any number base works just as well as 10, or 5, or 2. Many junior high school youngsters who have never before found an interest in math are now computing in base five and base seven with great enthusiasm. And, as a result, they have more understanding and appreciation of the decimal system. Perhaps you would be interested in discussing other number bases with your youngster.

Of What Use Are Sets?

If your child is one of the fortunate youngsters who are attending a school with a modernized mathematics program, he will have become acquainted with the notion of a set early in his elementary experience, and the farther he goes, the more he will use such descriptive words as "sets," "operations," and "relations." In the early elementary years, the notion of a set will provide him with a way of counting and classifying objects, grouping them, and manipulating number symbols. Even his preschool experience will have provided him with some background in this regard. He will already be familiar with a set of dishes or a set of building blocks.

In his early school years, this notion will provide him with a definite anchor to which he will be able to tie many other abstract ideas. During the upper elementary years, the idea of sets will help him to think more clearly about certain parts or phases of his mathematics. He will en-

vision the counting numbers, zero, fractions, decimals, negative numbers, odd numbers, even numbers, and primes as definite parts or subsets of the rational number system. Perhaps this sounds somewhat irrelevant to you, yet such ideas assist the child immeasurably in systemizing and classifying mathematical subject matter.

In the intermediate and upper elementary years, a modern mathematics text develops operations in connection with sets. A student will deal with the union and intersection of sets, the empty or null set, and one-to-one correspondence relationships. These are examples of mathematical ideas centered around sets. This will mean that the student is becoming more sophisticated in his use of set ideas and is learning to classify objects and events in their proper perspective.

As he progresses through school, he will be using these terms in reference to his algebra, geometry, trigonometry, and more advanced work. In algebra the set ideas and notation will assist him in describing and thinking through certain problem situations. You may hear him say things like: "I am trying to find the set of all x's such that the sentence, x plus 4 is equal to 10 minus 2x, is true." You perhaps feel that this is a rather complicated way of talking about the simple algebraic relation $x+4=10-2x$. But the student should not be confused because he has never heard the relation expressed in any other way. After all, his description tells considerably more about the true mathematical situation than would be expressed by simply saying: "What's x in the problem $x+4=10-2x$?"

You may also hear your youngster speak of a line as containing an infinite set of points, or say that the intersection set of a given line and a circle contains two points. There are many applications of set ideas in geometry and trigonometry. The set concept is important in contemporary mathematics. As a matter of fact, a large portion of mathematics can be developed from the notion of sets and operations thereon. It may be said that set notions, operations, and applications are the threads from which the fabric of contemporary mathematics is woven.

When your youngster enters college he will very likely be required, regardless of the field he chooses, to take one or more mathematics courses. Here again the set notions will stand him in good stead. If he chooses to major in science or mathematics, as more and more college students are now doing, he may take advanced courses which will deal with set theory exclusively.

These pages of explanation may have served to give you some idea of a few of the elementary concepts and the methods of teaching them which are typical of the mathematics your youngster is learning. But we have only scratched the surface. If this has evoked some enthusi-

asm on your part, you may be interested in delving more deeply into some of these topics. In the suggested reading list which appears at the end of this bulletin you will find books which provide more information on these and similar topics. Or you may be interested in taking a do-it-yourself course emphasizing these recent trends in mathematics education. Such courses are commercially available under the general name programed instruction. This is often referred to as machine teaching or automated instruction. A great many self-instructional programs are available in mathematics. You might enjoy working through one of these programs on your own. If you select a program dealing with modern mathematics, you will become familiar with two educational developments simultaneously: The recent trends in mathematics instruction and the recent developments in programed instruction.

"Miss Blake if we'd organize this like an assembly line, we'd turn out twice as many paper chains and cut working hours, too."

Courtesy: New York News Syndicate, Inc.

Programs Resulting in Improvements

EVIDENCE THAT traditional textbooks in mathematics lag behind the needs of our times has stimulated a number of curriculum improvement programs. In fact, there are so many experimental projects in teaching mathematics at both elementary and secondary levels that the uninformed person may be bewildered by the many labels and symbols (Madison, Suppes, University of Illinois Committee on School Mathematics, School Mathematics Study Group, Ball State Teachers College Program, University of Maryland Mathematics Project, University of Illinois Arithmetic Project, Boston College Mathematical Series, Greater Cleveland Mathematics Project, to name but a few). Most of the programs have much material in common, and some interesting findings about what children learn and the way in which they learn are discussed.

The experimental programs in many instances have motivated schools and school systems to examine critically their mathematics courses. This has led to new instructional programs in mathematics. In addition, commercial publishing companies are now using the results of the experimental programs. In fact some of the programs which started as experimental are now, after testing and alteration where necessary, being published in final form.

Madison Project

The Madison Project resulted from a conference between the principal of Madison Junior High School in Syracuse, N.Y., and Dr. Robert Davis of Syracuse University, to consider what could be done to revitalize a mathematics program which seemed dull and static. Starting with work in the seventh grade in 1957, Dr. Davis evolved his plan step by step, and then proceeded to experiment at lower and lower grade levels. After using his materials with a third grade class for several weeks, he was on the verge of declaring this level too low when suddenly the ideas began to evoke a response from the children and the enthusiasm started to run high. More recently, results with second grade children have

been amazing. The Madison first course consists mainly of fundamental concepts of arithmetic, algebra, and coordinate geometry which are usable at various grade levels, depending primarily on the teacher's skill and on the ability and interest of the pupils. The materials are intended to supplement a good arithmetic program. After the student has completed the materials provided by the Madison Project, he is prepared to study the School Mathematics Study Group First Course in Algebra or other similar courses.

Suppes and Hawley Elementary Mathematics Projects

At Stanford University, Professors Patrick Suppes and Newton Hawley experimented with teaching geometry in the first grade in 1957 and 1958. Using student groups of which their own daughters were members, they demonstrated that first-graders can understand and enjoy geometric concepts. This experimentation with the introduction of geometric concepts and methods at a much earlier age than customary in American schools had positive results that argued strongly for the possibility of enriching the present elementary school curriculum with geometry. The Carnegie Corporation of New York gave a grant to finance the publication of a geometry book and a teacher's manual for use in primary grades, and the National Science Foundation is providing support for continuing experimentation in this area.

Further experimentation at Stanford University seeks to modernize and improve the mathematics curriculum in grades 1 through 6. This is called the Suppes Arithmetic Project and involves an approach to number and arithmetical operations through the concept of sets and operations on sets. The combining of sets of physical objects is a more concrete operation than the summing of numbers. The early introduction of sets permits a clear, simple, and precise delineation of numbers as properties of sets. This covers all the usual objectives of primary arithmetic and introduces additional concepts not ordinarily found in many standard textbooks.

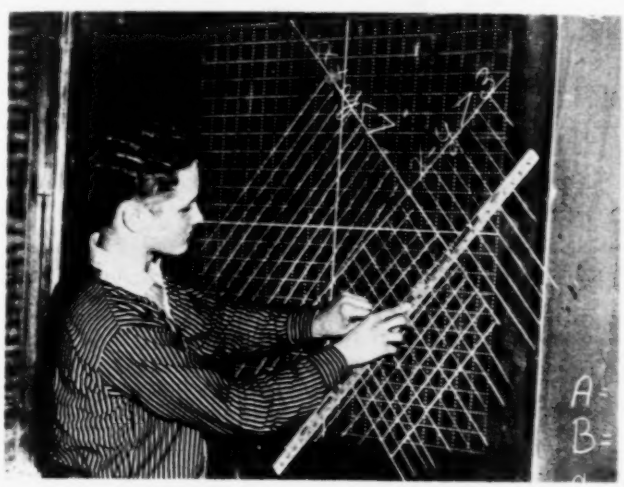
University of Illinois Committee on School Mathematics

As early as 1952 the University of Illinois Committee on School Mathematics was working on materials of instruction, the development of teaching methods, and the preparation of teachers for a new curriculum in mathematics for the secondary schools. The UICSM textbooks, which have been used experimentally by more than 10,000 students, emphasize consistency, precision of language, structure of mathematics, and understanding

through discovery. Students, led by teachers, discover and generalize for themselves. Materials for grades 9 through 12 for both students and teachers have been developed.

School Mathematics Study Group

The School Mathematics Study Group was established in 1958 by the American Mathematical Society in consultation with the National Council of Teachers of Mathematics and the Mathematical Association of America and has been financially supported by the National Science Foundation. Its work represents an extensive united effort and combines the thinking of many people from all segments of the mathematics profession. Sample textbooks have been written by teams composed of mathematicians and teachers selected for this specific job. Materials from grades 4 through 12 have been tested with thousands of students in selected experimental centers and have been revised on the basis of test findings. Study guides, aimed at broadening the mathematical background of teachers, have been produced. Programed learning texts of some SMSG materials have been prepared. This programing method of instruction presents subject matter in a printed sequence designed to be so easy to follow that students can study almost without supervision and at their own rate. SMSG headquarters, formerly at Yale University, is now located at Stanford University, Palo Alto, Calif.



Courtesy: Maplewood-Richmond Heights High School, Maplewood, Mo.

Mapping the intersection of two sets. A meaningful way of describing the solution set of the inequality system.

Commission on Mathematics

The report of the Commission on Mathematics of the College Entrance Examination Board, written in 1959, has to do chiefly with the college-preparatory mathematics program for grades 9 through 12. Its recommended modifications of the traditional content include: (1) An introduction to methods of deductive reasoning in algebra; (2) The reduction of the number of theorems, the introduction of coordinate geometry, and the incorporation of solid or space geometry in traditional plane geometry; (3) A treatment of trigonometry which will emphasize contemporary applications rather than the solution of triangles by logarithmic computations; and (4) An introduction to statistical thinking (or modern algebra) at the senior-year level in the secondary school. The influence of the Commission's report is evident in more than one of the experimental programs.

Ball State Teachers College Experimental Program

Major emphases of the Ball State Teachers College Experimental Program are on the axiomatic (or self-evident truth aspect) structure of mathematics and on precision of language. This program is planned for students in grades 7 through 12. Ball State textbooks, which are already available, are characterized by the careful attention given to logical development.

"Dad's tax accountant is coming over this evening. Suppose I take this problem home and watch how he tackles it."

Courtesy: New York News Syndicate, Inc.

University of Maryland Mathematics Project

The University of Maryland Mathematics Project (commonly referred to as UMMaP) has designed and produced text materials of a modern type suitable for use in the seventh and eighth grades. Experiments were conducted to determine what mathematics can be taught at these levels.

Other Programs

The Boston College Mathematical Series, which will provide materials for grades 8 through 12, started as an inservice program for mathematics teachers seeking more information on modern mathematics. The University of Illinois Arithmetic Project was granted funds in 1958 by the Carnegie Corporation for a 5-year experiment in arithmetic. The Greater Cleveland Mathematics Program (GCMP) spurs interest at the elementary school level. Countless efforts at improvement are being made by local school groups and individual dedicated teachers in their own classrooms. Thousands of students and hundreds of teachers are trying materials to see what works, and millions of dollars are being expended to good effect. Of these experimental programs each has its own unique features, and all have common elements and an identical final objective: the improvement of the mathematics curriculum and the acceleration of needed changes therein.

The foregoing listing of experimental work going on in mathematics education is by no means all-inclusive. It is simply a listing of some of the projects to give you, the parent, an idea of the time and effort which is being expended throughout the Nation to provide your youngster with the opportunity of studying a type of mathematics which is timely in content and interesting in method. This experimentation and the evaluation and revision of instructional materials in mathematics have resulted in new offerings for the youth of today. Much material is already available. The challenge is now to the school administrator, to the teacher, to you, and to your child.

Federal Aid for State and Local Programs

The 85th Congress of the United States recognized the need to improve school mathematics when in 1958 it passed the National Defense Education Act (NDEA). Title III of this act authorizes payments on a matching basis to State educational agencies for:

> (1) The acquisition of laboratory and other special equipment, including audiovisual materials and equipment, and printed materials (other than textbooks) suitable for use in providing education in science, mathematics, and modern foreign languages in public elementary and secondary schools or both.
>
> (2) Minor remodeling of the laboratory or other space used for such materials or equipment.

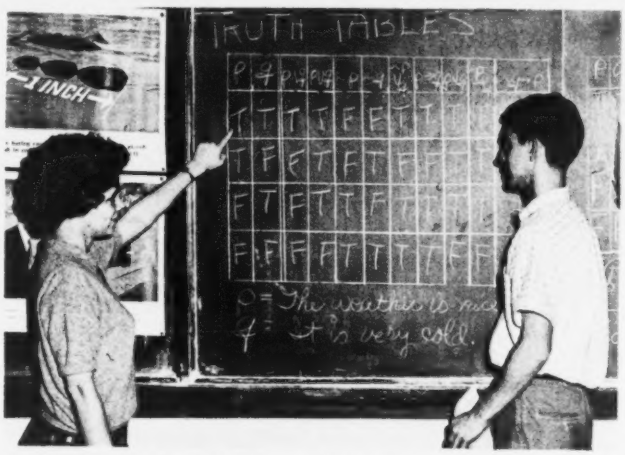
(3) The expansion or improvement of State supervisory or related services in the fields of science, mathematics, and modern foreign languages.

Local school districts can improve their mathematics instruction with NDEA funds after their State department of education has approved an improvement project in accordance with the provisions of State plans, standards, and principles of priorities. (Loans are available to nonprofit private schools for the same purposes.)

For the first 6 years of NDEA, ending June 30, 1964, the States received matching funds for 78,760 projects involving specialized mathematics equipment and materials. These projects cost an estimated $51,023,319 in Federal and State or local matching funds. During the same period of time approximately $6 million in Federal and local funds were expended to improve mathematics supervisory and related State services rendered to the local schools. In spite of all this, the needs in this area are still far from completely met.

At the date of the passage of the NDEA, only 6 States had special supervisors in the field of mathematics. Six years later, 46 States reported that they were providing supervisory services in mathematics. These supervisors have encouraged the use of new curriculum materials and have arranged inservice programs to prepare mathematics teachers to teach the new materials. Many teachers also continue to improve and modernize their instruction by attending National Science Foundation Institutes.

Thus, as you see, many influences are at work to improve the mathematics program your child is studying. If the influences have not yet been felt in your community, then perhaps you, the interested parent, will wish to consult the principal and teacher to discuss the merit of such an instructional program of mathematics.



Courtesy: Maplewood-Richmond Heights High School, Maplewood, Mo.

Interesting mathematics from an experimental program.

Courtesy: Valley High School, Albuquerque, N. Mex.

Interest in mathematics has been renewed with the installation of specially equipped mathematics laboratories such as this one.

How Can Parents Help?

TODAY'S TECHNOLOGICAL ADVANCES have opened up numerous opportunities for the girl or boy with mathematical ability. No one can predict how many times these opportunities will have multiplied by the time your child finishes school. If your child shows an interest in mathematics, encourage him and cultivate his enthusiasm.

1. Modern mathematics challenges your child to think and gives him a chance to think. Let him rely on his powers of reasoning. If he seems to be doing math in a different way, refrain from saying, "That isn't the way I did it." Try rather to understand his way and relate it to how you did it.

2. Set aside a convenient place and time for your child to study. Continuity between concrete and abstract factors in mathematics requires a quiet atmosphere conducive to concentration.

3. Join a modern mathematics class for adults, if one is offered in your community. This is one way of keeping up with your child in school. There is such a demand for modern mathematics in some areas that additional classes have had to be scheduled.

4. Consult the mathematics supervisor of your local school system or of your State concerning any information or assistance you need. The guidance counselor of your local school will have information and materials which will be helpful.

5. When you visit your child's school see whether the mathematics materials and equipment are adequate and up to date. Ask the teacher or principal if the school has taken advantage of the National Defense Education Act to provide modern mathematics teaching and learning materials for the school.

6. Let your youngster's teacher know you are interested in the modern trends in mathematics education. Ask if such developments are included in your child's course.

7. You may be interested in providing some learning materials at home, in addition to books, for your child's mathematics education. You may wish to ask his teacher for advice on the type and kind to purchase. There are many learning kits available for mathematics study.

8. Mathematics offers many recreational and pastime possibilities for life in the home. You may promote activity on these lines by participating with your youngster and other members of the family in puzzle problems, computational trick problems, and mental arithmetic.

9. If your child has a natural talent for mathematics, begin early to make college plans for him. Talk to guidance counselors and consult college catalogs. Be sure he takes the courses in high school which will prepare him adequately for the college of his choice. The mathematics required for college entrance cannot be mastered overnight.

10. Encourage your child to read recreational books and view television programs which have scientific or mathematical implications. More and more TV programing is being devoted to scientific or mathematical developments. The suggested reading list which appears on page 28 contains names of books and pamphlets which provide further information on any of the topics which have been introduced in this publication. The majority of these materials have been written for popular reading and do not presuppose a preparation in mathematics.

Courtesy: Marion High School, Marion, Va.

Hard at work in a classroom section of a mathematics laboratory, this student has the stimulating advantages of concept building devices, precision instruments, up-to-date references, and other modern equipment.

11. If your child is mathematically inclined, then your course of action is evident. If your child is not mathe-

matically inclined and does not anticipate a career in which a great amount of mathematics would be needed, he may still need to know something of the new developments and the much expanded usage of mathematics. Point out through current events how mathematics contributes to outstanding achievements. Stress the ever-expanding vocational possibilities. If you are unfamiliar with the trends in mathematics education which your youngster is experiencing, ask him to give you an explanation. Even if you are familiar with these trends, request an explanation. This usually results in a good learning experience for the student, since he is temporarily placed in the teaching role. In any case, he is made aware of your interest in his new mathematics. Also, if you have not made a conscientious effort to keep abreast of current developments in mathematics education you will find this a most rewarding educational experience.

The importance of your role in the education of your child cannot be overemphasized. The part which your child will assume in the development and assimilation of tomorrow's culture will in large measure be the result of his interests, attitudes, and intellectual endeavors of today. On your shoulders rests the responsibility—and the great privilege—of helping, through your child, to fashion that future. Your contribution may be more significant than you think.

"Billy's gonna be a rocket scientist some day. He can already count to 10 backwards."

Courtesy: New York News Syndicate, Inc.

Suggested Reading List

ADLER, IRVING. *The New Mathematics.* New York, The John Day Co., 1958. 187 p. (Also available in paperbound edition.)

BAKST, AARON. *Mathematics: Its Magic and Mastery.* Princeton, N.J., Van Nostrand Co., 1952. 790 p.

BELL, E. T. *Mathematics: Queen and Servant of Science.* New York, McGraw-Hill Book Co., 1951. 437 p.

BOEHM, GEORGE A. W., *and others. The New World of Math.* New York, Dial Press, 1959. 128 p.

JOHNSON, DONOVAN A., *and* GLENN, WILLIAM H. *Exploring Mathematics on Your Own.* St. Louis, Mo., Webster Publishing Co., 1960–62. (A set of 18 booklets)

LASS, ABRAHAM H. *How To Prepare for College.* New York, Affiliated Publishers, Division of Pocket Books, Inc., 1962. 466 p.

LIEBER, L. R., *and* LIEBER, H. G. *The Education of T. C. Mits.* New York, Norton Co., 1944. 230 p.

MOTT-SMITH, GEOFFREY. *Mathematical Puzzles for Beginners and Enthusiasts.* New York, Dover Publications, 1955. 248 p.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 1201 Sixteenth Street, NW., Washington 6, D.C. *Careers in Mathematics.* Washington, D.C., The Council, 1961. 28 p.

——— *The Growth of Mathematical Ideas* (Grades K–12). Twenty-fourth Yearbook. Washington, D.C., The Council, 1959. 507 p.

——— *The Revolution in School Mathematics.* Washington, D.C., The Council, 1961. 90 p.

PEDOE, DANIEL. *The Gentle Art of Mathematics.* English Universities Press, 1958. *Order from* The Macmillan Co., New York. 143 p.

SAWYER, W. W. *Prelude to Mathematics.* Baltimore, Md., Penguin Book Co., 1955. 214 p. (Paperbound)

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE, OFFICE OF EDUCATION. *Careers in Engineering, Mathematics, Science, and Related Fields: A Selected Bibliography,* by A. Neal Shedd *and others.* Washington, U.S. Government Printing Office, 1961. 39 p. (Bulletin 1961, No. 8, OE–26007.)

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